



Original article

## Very brief on quantum computing with special respect to ion traps technology

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### ABSTRACT

*Because of the continuing miniaturization of integrated circuits it appears that quantum phenomena will play a more and more dominant role in their design and functioning. Therefore, the work exposes concise quantum mechanical and mathematical background of quantum phenomena based computing through the introduction of concepts such as Hilbert space, qubit, Bloch sphere, quantum gate and measurement. Recent improvements in ion-trap technology based computing suggest ion-trap technology as the most promising one for use in quantum computing. So, the paper describes the ion-trap based technology, its use in quantum computation and its latest applications in quantum computing. It turns out that recent ion-trap technology based computers dramatically improves on all of the Vincenzo's criterions.*

**Key words:** miniaturization; Hilbert space; qubit; Bloch sphere; measurement; ion-trap technology.

### 1. INTRODUCTION

As early as 1965, Gordon Moore noticed that the number of components that could be placed on a chip had grown exponentially over many years, while the feature size had shrunk at a similar rate [1].

In 2008 some of Intel processors were based on 45 nm lithography and in 2021 on 14 nm [2].

It seems that the feature size will soon become smaller than some less well defined limit, where the electrons that do the work in the semiconductor devices, will start to show that their behaviour is governed by quantum mechanics, rather than the classical physical laws that are currently used to describe their behaviour. The classical description of the operation of semiconductor devices will become impossible when the feature size reaches the coherence length. This quantity depends on the details of the material, the processing and the temperature at which the device operates, but typically is in the range of a few nanometers to some tens of nanometers [3].

So, it is clear that the progress that we have today will soon lead to a situation where it is no longer possible to describe the flow of electricity as a classical current. While a classical device, such as the workhorse FET, requires a continuous relationship between current and voltage, this

will no longer be the case in the quantum mechanical regime, [3].

At the one-atom-per-bit level and, realistically, even a little before this, it will be necessary to use quantum effects to read bits from and write bits to the memory register of an ultra-small computer. So even on memory grounds alone, there is a strong reason to investigate the operating principles and feasibility of quantum devices [4].

In addition to the exponential improvement in transistor density and clock speed, there has been a concomitant improvement in energy efficiency. The early computers generated relatively large amounts of heat per logical operation. As computer components have become smaller, and transistor density per chip has increased, the components have had to be made more energy efficient per logical operation to avoid thermal damage to the semiconductors during normal operation. The trend suggests that computers will soon reach the 1 kT level (where k is Boltzmann's constant  $k = 1.3805 \cdot 10^{-23} \text{ JK}^{-1}$  and  $T \approx 300 \text{ K}$  at room temperature), which is the typical amount of energy in thermal noise at the atomic level. Consequently, 1 kT marks a practical order of magnitude threshold for controllable quantum devices. At this point every aspect of computer operation, from loading programs, running such programs, and reading the answers

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will be dominated by quantum effects. Even the design of algorithms will have to be rethought to make the best possible use of the quantum possibilities, [4].

## 2. FROM CBIT TO QUBIT

A classical bit (cbit) is the smallest unit of digital data and is limited to the two discrete binary states, 0 and 1. A quantum bit (qubit) can additionally enter a superposition of states, in which the qubit is effectively in both states simultaneously. The qubit can be considered as a representation of the pure Hilbert state space of a 2-level quantum mechanical system, which is described in Dirac's 'bra-ket notation' by the state

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \tag{1}$$

where  $\alpha$  and  $\beta$  are complex numbers satisfying the equation  $|\alpha|^2 + |\beta|^2 = 1$ ; such that measurement would result in state  $|0\rangle$  with probability  $|\alpha|^2$  and  $|1\rangle$  with probability  $|\beta|^2$ . Formally, a qubit is represented in the 2D complex vector space, where the  $\alpha|0\rangle + \beta|1\rangle$  can be represented

in the standard orthonormal basis as for  $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  the ground state and for  $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  the excited state, or on the Bloch sphere as in Fig. 1, [5].

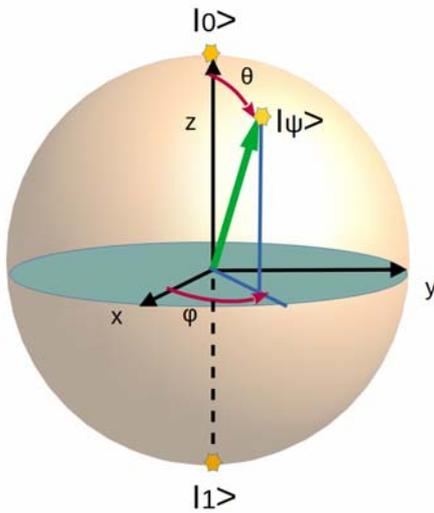


Fig. 1 Bloch sphere representation of a qubit

An especially practical form of a Bloch sphere equation is:

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi} \sin(\theta/2)|1\rangle \tag{2}$$

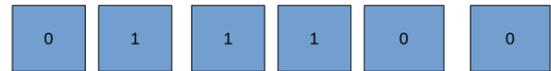
where  $0 \leq \theta \leq \pi$  and  $0 \leq \phi < 2\pi$ .

While a classical register made up of  $n$  binary bits can contain only one of  $2^n$  possible numbers, the corresponding quantum register can contain all  $2^n$  numbers simultaneously, (Fig.2). Thus, in theory, a

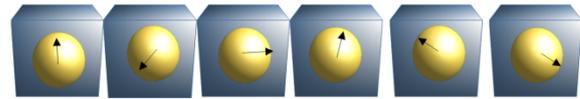
Quantum Computer (QC) could operate on seemingly infinite values simultaneously in parallel, so that a 30-qubit Quantum Computer would be comparable to a digital computer capable of performing  $10^{13}$  (trillion) floating-point operations per second (TFLOPS) which is comparable to currently fastest supercomputers [5].

In addition to the superposition, quantum information obeys other very unique properties:

- Destructive measurements: Upon reading the value of a qubit, a bit of classical information is obtained with certain probability, and any other bit states that were in the superposition along the measured state are lost.
- No-cloning: In general, quantum information cannot be copied.
- Entanglement: The measurement of a qubit may affect the state of other qubits. Alternatively, knowledge of the individual states does not yield a complete picture of the whole system [6].



CLASSICAL REGISTER – CAN CONTAIN ONLY ONE VARIATION OF 0 AND 1



QUANTUM REGISTER – CAN CONCURRENTLY CONTAIN ALL VARIATIONS OF 0 AND 1

Fig. 2 Quantum and classical register comparison

## 3. QUANTUM COMPUTATION

Like any other quantum state, the state of a qubit evolves under the influence of its Hamiltonian  $H$ . The time-dependent Schrodinger equation

$$i\hbar \partial / \partial t |\psi\rangle = H|\psi\rangle \tag{3}$$

has the solution

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle \tag{4}$$

If the Hamiltonian is time independent,

$$U(t) = \exp(-i(H/\hbar)t) \tag{5}$$

The evolution of quantum states can also be described using the compact notation

$$|\psi\rangle \xrightarrow{Ht} U|\psi\rangle \tag{6}$$

Since  $H$  is Hermitian, the evolution operator  $U$ , usually called the propagator, must be unitary. It is stressed that the previous discussion supposes the Hamiltonian is time-independent, that is, it does not vary with time. This will not be true in a quantum computer, which is controlled by

varying the Hamiltonian. In many cases, however, the Hamiltonian is piecewise constant, that is it has a constant value for some finite length of time, and is then replaced by a different constant value for another finite time period, and so on. In this case the evolution can be described using a series of propagators

$$|\psi\rangle \xrightarrow{H_1 t_1} \xrightarrow{H_2 t_2} \xrightarrow{H_3 t_3} U_3 U_2 U_1 |\psi\rangle \quad (7)$$

with  $U_1 = \exp[-i(H_1/\hbar)t_1]$  and so on, (Fig.3). The situation is much more complicated when Hamiltonian varies continuously with time; it is possible to write down a formal solution of the form of equation (7), but this is not generally a useful approach [7].

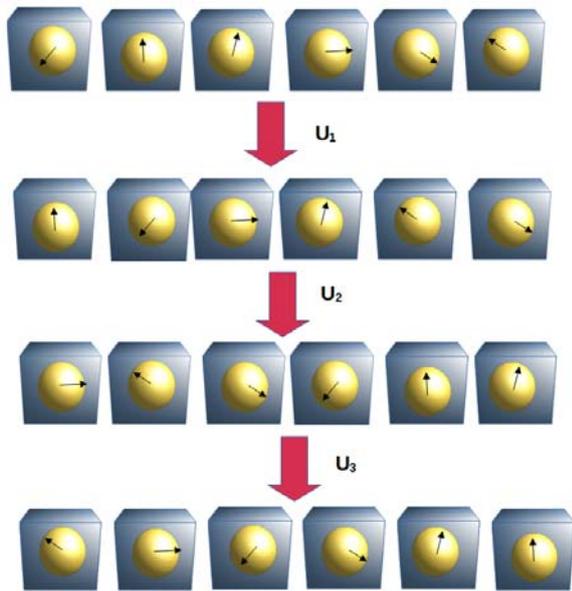


Fig. 3 Quantum computation

The fact that any propagator describing the evolution of a quantum system is unitary has several significant consequences. Firstly it means that every propagator has an inverse, and so quantum evolution is reversible. That is, there is not loss of information. One exception to this general principle is measurement, which is discussed in more detail below. Secondly unitary transformations are length preserving and can in general be thought of as rotations of the vector describing the quantum state. Since qubits reside on the Bloch sphere, the evolution of an isolated qubit under any Hamiltonian corresponds to a rotation of the vectors on the Bloch sphere [7]. The fundamental idea of quantum computing is that information is stored in quantum bits and processed by quantum logic gates. Just as classical logic gates take classical bits from one state to another, so quantum logic gates take qubits from one state to another. This can be achieved by modifying the system's Hamiltonian, by applying additional control fields to the background Hamiltonian which underlies the system. Applying Hamiltonians will cause qubits to evolve under unitary transformations, which are reversible [7].

Measurement of a quantum state changes the state. If a state  $|v\rangle = a|u\rangle + b|u_\perp\rangle$  is measured as  $|u\rangle$ , then the state  $|v\rangle$  changes to  $|u\rangle$ . A second measurement with respect to the same basis will return  $|u\rangle$  with probability 1. Thus, unless the original state happens to be one of the basis states, a single measurement will change that state, making it impossible to determine the original state from any sequence of measurements, [8]. That is, there is a loss of information, (Fig.4).

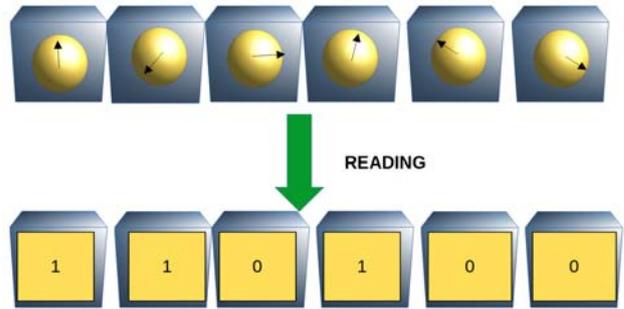


Fig. 4 Reading of a quantum register

#### 4. QUANTUM GATES AND CIRCUITS

In a classical computer, a gate is a hardware device that executes a defined operation on a cbit. In a quantum computer, a gate is a quantum mechanical operation on the qubit wave function specified by a Hamiltonian function. As is known by now, the Hamiltonian describes the energy of the quantum system as a function of coordinates such as position, momentum, angular momentum, and sometimes time. The application of defined external fields, which are applied according to a Hamiltonian for a specified time to the system, execute a particular computational operation on the qubit wave function. Since the fields are external to the qubit system, the energy of this system can be changed [9].

If there are  $n$  Cbits, each representing either 0 or 1, the state of each can be found just by looking. In harsh contrast, if there are  $n$  Qubits in a superposition (8) of computational basis states, there is nothing whatever that can be done to them to extract from those Qubits the vast amount of information contained in the amplitudes  $\alpha_i$ .

$$|\Psi\rangle_n = \sum_{i=0}^{2^n} \alpha_i |i\rangle_n \quad (8)$$

One cannot read out the values of those amplitudes, and therefore cannot find out what the state is. The state of  $n$  Qubits is not associated with any ascertainable property of those Qubits, as it is for Cbits. There is only one way to extract information from  $n$  Qubits in a given state. It is called making a measurement. The process of measurement (reading) is carried out by a piece of hardware with a digital display, known as an  $n$ -Qubit measurement gate ( $M_N$ ), as is schematically described in Figs. 5, 6 and 8.



Fig. 5 Measurement

In contrast to unitary gates, which have a unique output state for each input state, the state of the Qubits emerging from a measurement gate is only *statistically* determined by the state of the input Qubits. In further contrast to unitary gates, the action of a measurement gate cannot be undone: given the final state  $|i\rangle$ , there is no way of reconstructing the initial state  $|\Psi\rangle$ . Measurement is irreversible. Nor is the action of a measurement gate in any sense linear. It can be shown that  $n$ -Qubit measurement gates can be realized by applying 1-Qubit measurement gates to each of the  $n$  Qubits. The process of measurement can thus be reduced to applying multiple copies of a single elementary piece of hardware: the 1-Qubit measurement gate [10].

In the classical circuit model, circuits are networks composed of *wires* that carry bit values to *gates* that perform elementary operations on the bits. In the quantum circuit model, there are logical qubits carried along ‘wires’, and quantum gates that act on the qubits. A quantum gate acting on  $n$  qubits has the input qubits carried to it by  $n$  wires, and  $n$  other wires carry the output qubits away from the gate. A quantum circuit is often illustrated schematically by a circuit diagram as shown in Figs 6, 7 & 8. The wires are shown as horizontal lines, and we imagine the qubits propagating along the wires from left to right in time. The gates are shown as rectangular ‘U’ blocks. The gates come from some finite family, and they take information from input wires and deliver information along some output wires.

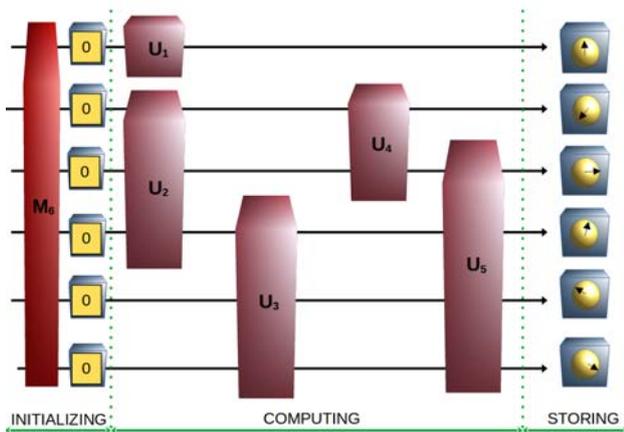


Fig. 6 First step in quantum computation

In quantum computing, we refer to a unitary operator  $U$  acting on a single-qubit as a 1-qubit (unitary) gate. We can represent operators on the 2-dimensional Hilbert space of a single qubit as  $2 \times 2$  matrices. The not gate is often identified with the symbol  $X$ , and is one of the four Pauli gates [11]:

$$\sigma_0 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_1 = \sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \sigma_y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

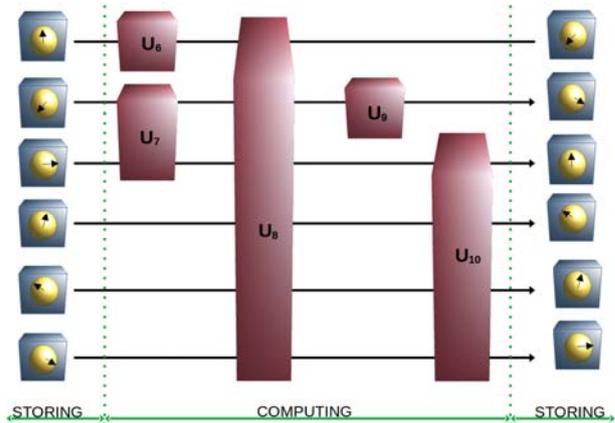


Fig. 7 An intermediate step in quantum computation

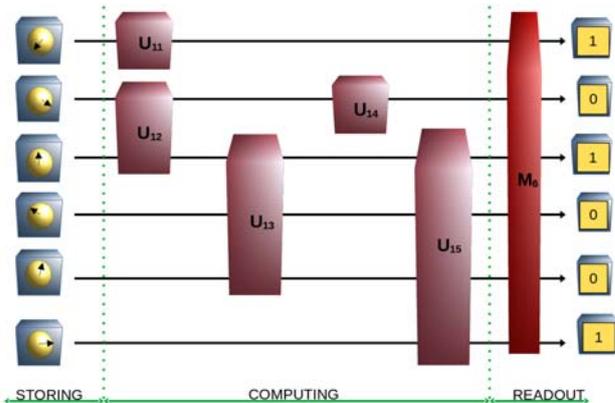


Fig. 8 Final step in quantum computation

Another important single-qubit gate is *Hadamard* gate  $H$ :

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

In circuit notation, one-qubit gates can be represented as in Fig.9.

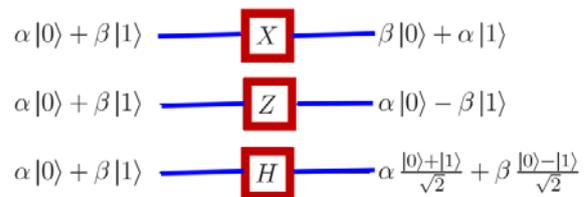


Fig. 9 Circuit notation of one- qubit gate

An arbitrary quantum computation on any number of qubits can be generated by a finite set of gates that is said to be *universal* for quantum computation. Any unitary matrix specifies a valid quantum gate! The interesting implication is that in contrast to the classical case, where only one non-trivial single bit gate exists – the NOT gate – there are many non-trivial single qubit gates [12]. The prototypical multi-qubit quantum logic gate is the *controlled- NOT* or CNOT gate. This gate has two input qubits, known as the *control* qubit and the *target* qubit, respectively. The circuit representation for the CNOT gate is shown in the Fig.10 [12].

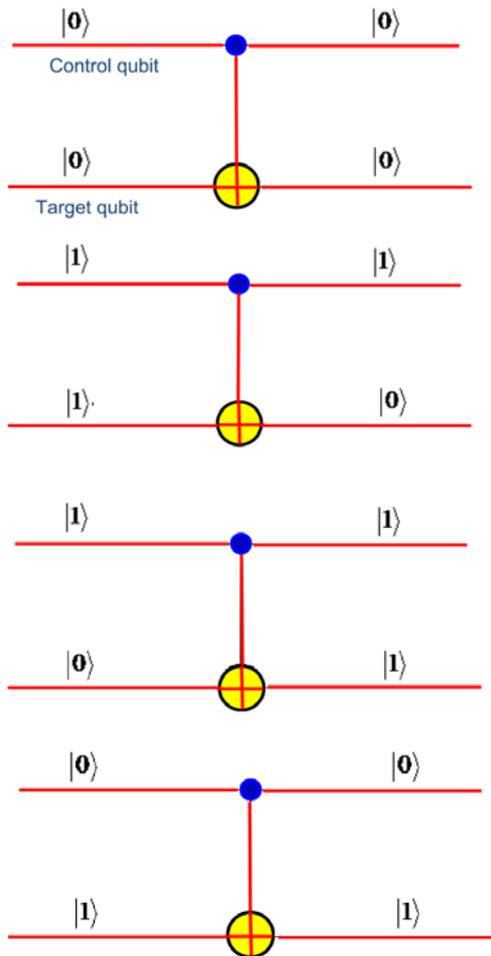


Fig. 10 CNOT gate

The top line represents the control qubit, while the bottom line represents the target qubit. The action of the gate may be described as follows. If the control qubit is set to 0, then the target qubit is left alone. If the control qubit is set to 1, then the target qubit is flipped, [12].

In a sense the controlled- and single qubit gates are the prototypes for *all* other gates because of the following remarkable *universality* result: *Any multiple qubit logic gate may be composed from CNOT and single qubit gates* [12].

The short answer to the question of whether or not there is a finite set of quantum gates that is universal is just “no.” However, even though it is impossible to have a

finite number of quantum gates that will generate every other possible quantum circuit, people have shown there is a finite collection of gates that can be used to approximate every possible circuit, all of the circuits that are needed can be constructed from above mentioned gates; five that act on just one qubit, and one, the *CNOT* gate, that acts on two qubits [13].

## 5. REQUIREMENTS FOR PRACTICAL BULK QUANTUM COMPUTING IMPLEMENTATION

Vincenzo enumerated what he felt were the major requirements for implementing practical bulk quantum computing [14]:

1. Physical scalability, allowing the number of qubits to be sufficiently increased for bulk implementation.
2. Qubits must be able to be initialized to arbitrary values.
3. Quantum gates that operate faster than the decoherence time.
4. A universal gate set for running quantum algorithms.
5. Qubits that can be easily read correctly.

## 6. ION TRAPS REALIZATION OF QUBITS

Atomic ions have some attractive properties for use as qubits: qubits can be defined in ways that make decoherence very slow while simultaneously allowing for readout with high efficiency. To avoid perturbing these ideal properties, the ions are best isolated in space [15]. This can be achieved with electromagnetic traps, which arrange electric and magnetic fields in such a way as to create a potential minimum for the ion at a predetermined point in space [16].

The Pauli trap (Fig. 11) consists of an axially symmetric set of electrodes. The electrodes on the symmetry axis have the same potential, while the ring has the opposite polarity. The resulting field is roughly that of a quadrupole, where the field vanishes at the center and increases in all directions. The voltage on the electrodes varies sinusoidally. The ion is therefore alternately attracted to the polar end caps or to the ring electrode. On average, it experiences a net force that pushes it towards the center of the trap. In the exact center, the field is zero and any deviation results in a net restoring force [16].

In 1995, Cirac and Zoller proposed to implement quantum computation which used laser beams to cool atomic ions confined in vacuum inside an ion trap, until they formed a stable linear ion array under the joint forces due to their mutual repulsion and the confining potential gradient. Since each cold ion of the stable array was identifiable, two internal electronic levels of each ion could act as a computational quantum bit, or qubit. A quantum gate, or quantum computational unit, requires a correlated action on at least two qubits. Cold ion internal level qubits have no direct coupling, since the separation of the ions in the linear array is large compared to atomic interaction

distances. However, since the minimum of the ion trapping potential is essentially harmonic, the ion array oscillates coherently in the trap at the centre of mass (CM) angular frequency  $\omega_{CM}$ , which remains the same, regardless of the number of ions. This and other quantized normal modes of oscillation of the ions in the trap can be cooled to their lowest energy state using laser beams, in order to act as a “motional qubit” to couple the ions on demand [19]. The Paul Trap can also be made into an extended linear trap ([17], [18]):

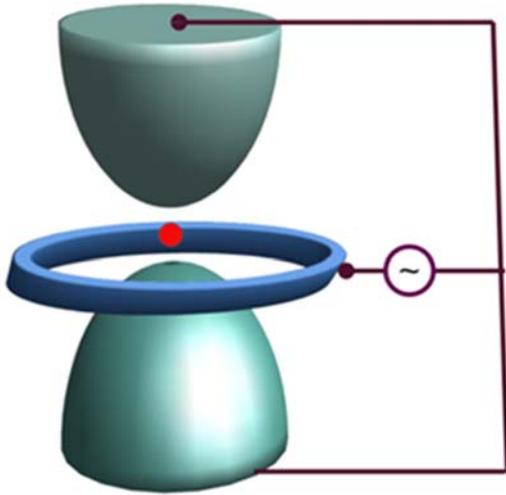


Fig. 11 The Paul trap

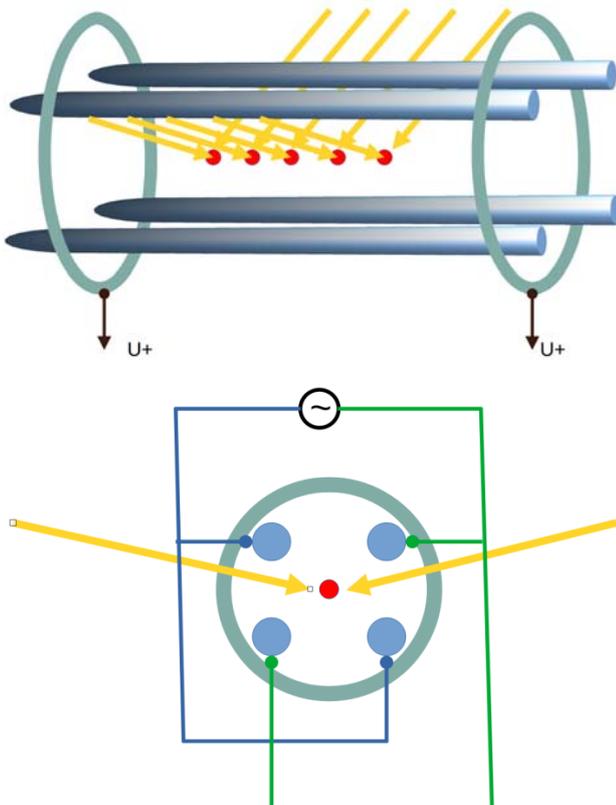


Fig. 12 Linear Paul trap

A single laser beam is split by beam splitters and acousto-optic modulators into many beam pairs, one pair illuminating each ion [20].

Figure 12 shows the geometry used in this design, which consists of four parallel rods that generate a quadrupole potential in the plane perpendicular to them. The quadrupole potential is alternated at a radiofrequency, and the time-averaged effect on the ions confines them to the symmetry axis of the trap, while they are free to move along this axis. A static potential applied to the end caps prevents the ions from escaping along the axis. Yellow arrows represent laser beams used for atom cooling [20].

## 7. THE DEVELOPMENT OF ION TRAPS COMPUTING

One of the reasons David Wineland in 2012 received Nobel Prize was that “Wineland and his group were the first to carry out experimentally a two qubit operation (the Controlled NOT gate, CNOT) between motion and spin for  $\text{Be}^+$  ions, ([21])” and “since then, the field of quantum information based on trapped ions has progressed considerably” [22].

“Controlled-NOT” quantum logic gate in conjunction with simple single-bit operations, forms a universal quantum logic gate for quantum computation [21].

In 2016 researchers at NIST entangled more than 200 beryllium ions [23].

In 2021 it is reported high-fidelity state readout of a trapped ion qubit using a trap-integrated photon detector [24].

Trapped ions have delivered world records in decreasing errors that occur in the operation of quantum gates used to carry out calculations ([25], [26], [27]).

In 2021 it is reported the realization of a microchip-based quantum computer incorporating an architecture in which calculations are carried out by shuttling atomic ions (quantum CCD design) [28].

Quantum computers are often characterized by how many qubits they can host. Without the ability to correct for unavoidable errors, the number of usable qubits is limited both by the magnitude of individual errors and by the accumulation of all the errors in the system. A parameter known as the quantum volume provides a measure of how many usable qubits a machine contains, based on the overall system performance [27].

Pino et al. determined that their device has a quantum volume of 64, which means that it can do generalized computations using up to 6 qubits. In principle, a single quantum-computing module built according to the quantum-CCD design could hold hundreds or even thousands of usable qubits. Therefore, it should be possible to scale up this architecture to a million-qubit machine using a modular approach ([27], [28], [29]).

A different trapped-ion architecture had previously been demonstrated in an alternative microchip-based quantum computer, one in which laser beams manipulate the internal state of individual ions in a chain of trapped stationary ions [30].

Trapped-ion technologies are gaining momentum in the quest to make a commercial quantum computer. Earlier this year, technology and manufacturing company

Honeywell launched its first quantum computer that uses trapped ions as the basis of its quantum bits, or ‘qubits’, which it had been working on quietly for more than a decade. Honeywell, headquartered in Charlotte, North Carolina, is the first established company to take this route, and it has a 130-strong team working on the project. In October, seven months after the launch, the firm unveiled an upgraded machine; it already has plans to scale this up. And Honeywell is not the only company planning to make trapped-ion systems at scale. Last month, University of Maryland spin-off firm IonQ in College Park announced a trapped ion machine that could prove to be competitive with those of IBM or Google, although the company has yet to publish details of its performance. Smaller spin-off firms — such as Universal Quantum in Brighton, UK and Alpine Quantum Technology in Innsbruck, Austria— are also attracting investment for trapped-ion projects [31].

## 8. CONCLUSION

Theoretical model of quantum computing, except for initialization and readout, is based on time-dependent Schrodinger equation which preserves state information. Initialization and readout, generally, introduce loss of state information.

There is a universal set of quantum gates.

The physical realization of quantum computing is still in the development ([31]).

Quantum computing based on trapped ion technology has advanced in respect to all criteria posed by Vincenzo ([14]).

Pino and fellow workers’ quantum computer ([28]) constitutes an impressive achievement, yet again illustrating the coming of age of trapped-ion quantum computing as a leading hardware platform, [27].

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