



Original article

Fatigue endurance analysis of a surface stress raiser

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ABSTRACT

Fatigue-induced flaws play an important role in safety-integrity performances of large moving systems. Therefore, the present research work proposes a computational model to gain insight into such surface stress raisers. The analysis shows that a combination of damage-tolerance design with fracture mechanics-based concept is a key to generating relevant relationships between fatigue life and stress intensities.

Key words: *fatigue design, residual life, stress intensity analysis, surface flaw*

1. INTRODUCTION

Surface flaw phenomenon is one of the major deterioration mechanisms in existing large moving systems, causing considerable costs for maintenance and repair. In this context the development of reliable computational models to localize and characterize such part-through damage represented as quarter-elliptical and semi-elliptical crack is essential to assess the load bearing capacities of structural components.

Under cyclic loading, the stability of quarter-elliptical crack at a pin-loaded hole has been analyzed by Rigby and Aliabadi [1] using J-integral method and the boundary element method. Further, Yamashita et al. [2] have explored the driving mode caused by semi-elliptical crack through the Paris' crack growth concept together with the finite element method.

In order to evaluate the propagation of quarter-elliptical crack coupled with the pin-loaded effect Antoni and Gaisne [3] have suggested the analytical concept whereas Mikheevskiy et al. [4] have employed the crack growth concept proposed by Noroozi et al. [5]. The behaviour of the same failure-relevant configuration has been evaluated by Boljanović et al. [6] through the crack growth concept

proposed by Zhan et al. [7] and the J-integral method. Further, Boljanović et al. [8] have used the stress ratio dependant crack growth model [9] and the finite element method to assess the fatigue performances of quarter-elliptical corner crack at semicircular edge notch.

The present research work discusses the computational model recommended for fatigue assessments of plate-type configurations with quarter-elliptical crack-like flaw. Such fracture mechanics-based analytical tool is aimed at evaluating of structural responses and defining safety levels. Furthermore, several cases of technical interest are treated in detail, varying both the crack shape characteristics and parameters of loading profiles.

2. SURFACE FLAW PROGRESSION

At operating loads, surface flaws become sources of fatigue damages as a result of complex load-environment interactions. Therefore, the safety and long-term operation of large moving systems requires the inclusion of fracture mechanics-based concepts for analyzing the progression of these stress raisers [3-9]. In this context, the driving mode due to a quarter-elliptical crack (Fig.1) is herein

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theoretically examined through the crack growth model proposed by Walker [10], expressed as follows:

$$\frac{da}{dN} = \frac{C_A (\Delta K_A(a,b))^{m_A}}{(1-R)} \quad (1a)$$

$$\frac{db}{dN} = \frac{C_B (\Delta K_B(a,b))^{m_B}}{(1-R)} \quad (1b)$$

where da/dN , db/dN and a , b are the crack growth rate and crack length in depth and surface crack growth direction, respectively, R is the stress ratio, ΔK_A and ΔK_B are the stress intensity factors for two critical crack growth directions.

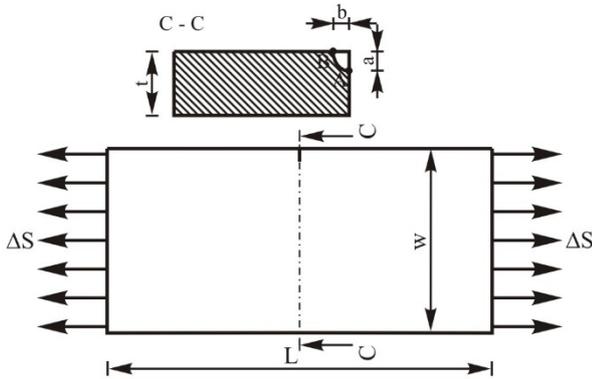


Fig. 1 Cyclically loaded plate with quarter-elliptical corner crack

In the frame of the fatigue endurance analysis the number of loading cycles N may be evaluated, integrating relevant crack growth rates (Eq. (1a) and (1b)) from initial a_0 , b_0 to final a_f , b_f crack lengths with respect to depth and surface crack growth direction, i.e.

$$N = \int_0^N dN = \int_{a_0}^{a_f} \frac{(1-R)}{C_A (\Delta K_A(a,b))^{m_A}} da \quad (2a)$$

$$N = \int_0^N dN = \int_{b_0}^{b_f} \frac{(1-R)}{C_B (\Delta K_B(a,b))^{m_B}} db \quad (2b)$$

3. STRESS INTENSITY EVOLUTION

The fatigue performance of quarter-elliptical corner crack (Fig. 1) is assessed employing the stress intensity factor [11], expressed as follows:

$$\Delta K = M_{qe} \Delta S \sqrt{\frac{\pi a}{Q}} \quad (3)$$

where ΔS and ΔK are applied stress range and stress intensity factor range, respectively, a is crack length in depth direction and Q represents the ellipse shape factor. Through the stress raiser evaluations the boundary conditions related to the crack shape, the crack size, thickness and width of the plate are taken into account by means of the correction factor M_{qe} , defined by

$$M_{qe} = \left(M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right) g_1 g_2 f_\phi f_w \quad (4)$$

$$M_1 = \sqrt{\frac{b}{a}} \left(1.08 - 0.03 \frac{b}{a} \right) \quad (5)$$

$$M_2 = 0.375 \left(\frac{b}{a} \right)^2 \quad (6)$$

$$M_3 = -0.25 \left(\frac{b}{a} \right)^2 \quad (7)$$

$$g_1 = 1 + \left(0.08 + 0.4 \left(\frac{b}{t} \right)^2 \right) (1 - \sin \phi)^3 \quad (8)$$

$$g_2 = 1 + \left(0.08 + 0.15 \left(\frac{b}{t} \right)^2 \right) (1 - \cos \phi)^3 \quad (9)$$

where b is crack length in surface direction, t and w are thickness and width of the plate, respectively, and ϕ is the angle location.

Further, the interaction between the ellipse crack shape effect ($a/b > 1.0$), and the effects of angle location and width of the plate are theoretically examined via the correction factors Q , f_ϕ and f_w , expressed as follows:

$$Q = 1 + 1.464 \left(\frac{b}{a} \right)^{1.65} \quad (10)$$

$$f_\phi = \left(\left(\frac{b}{a} \right)^2 \sin^2 \phi + \cos^2 \phi \right)^{1/4} \quad (11)$$

$$f_w = \sec \left(\frac{\pi b}{2w} \sqrt{\frac{a}{t}} \right)^{0.5} \quad (12)$$

4. FATIGUE DURABILITY EVALUATIONS

4.1 Residual life analysis

The present Section examines the fatigue performances of the plate with quarter-elliptical corner crack (Fig.1), made of 2024 T3 aluminium alloy. Such failure evaluations are performed adopting the applied maximum stress equal to $S_{max} = 310.63$ MPa and 240.79 MPa with $R = -1$, and the following geometrical and material parameters: $w = 76.2$ mm, $t = 2.3$ mm, $C_A = C_B = 1.2 \times 10^{-11}$, $m_A = 3.02$, $E = 73.1$ MPa, $\nu = 0.33$. The initial part-through crack is characterized by relevant lengths equal to $a_0 = b_0 = 20$ μ m in the case of depth and surface direction.

Under cyclic loading the stability of damaged plate is estimated through the stress intensity factor and residual life, using Eq. (3)-(12) and Eq. (2a)-(2b), respectively. The evaluated number of loading cycles, as a function of crack length in depth and surface directions, are shown in Fig.2a,

2b and Fig. 3a, 3b for maximum stress equal to $S_{max}=310.63$ MPa and $S_{max}=240.79$ MPa, respectively. In order to verify the life evaluations, experimentally tested data available in the literature [12] are reported in the same figures. By examining different comparisons, it can be deduced that safety-relevant results are in a quite good agreement for two critical directions.

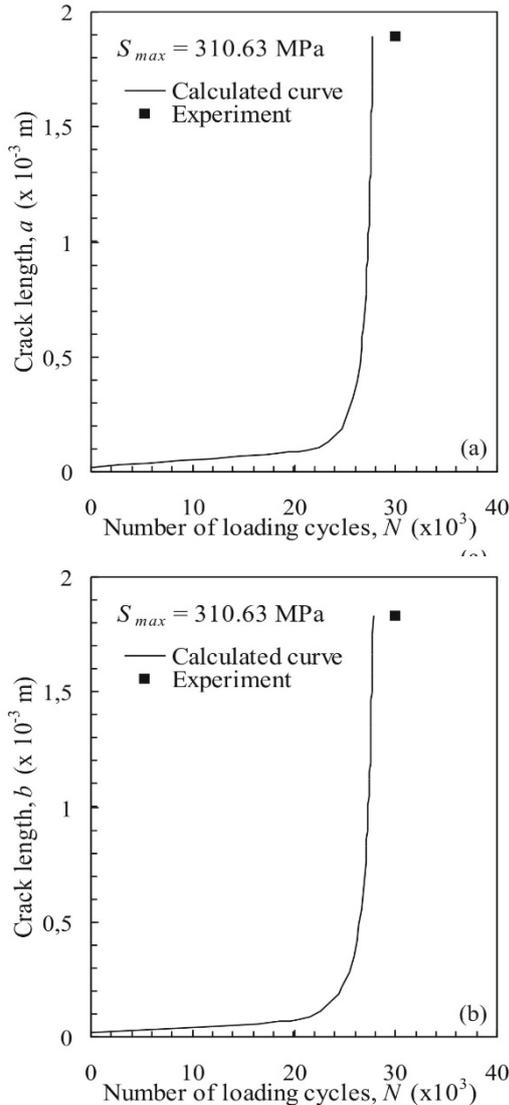


Fig. 2. Fatigue life evaluation for the plate with a quarter-elliptical crack ($S_{max}=310.63$ MPa): (a) $a-N$ and (b) $b-N$ (calculated curves are the present results and experiments are reported in Ref. [12])

4.2 Effects of stress ratio and thickness on the residual strength under cyclic loading

Now, through the stability evaluations the effect of applied load is examined in the case of surface corner flaws (Fig. 1). In this regard, the fatigue endurance of a plate made of 2024 T3 aluminium alloy (assuming the following applied maximum stress and geometrical parameters: $S_{max}=300$ MPa, $t=8$ mm, $w=50$ mm, $a_0=1.85$ mm, $b_0=2.1$ mm) is explored for three values of stress ratio ($R=0.1, 0.25$

and 0.625). According to the developed fracture mechanics-based analytical model, the driving mode due to quarter-elliptical crack is herein analysed through Eq. (3)-(12) and Eq. (2a)-(2b), respectively.

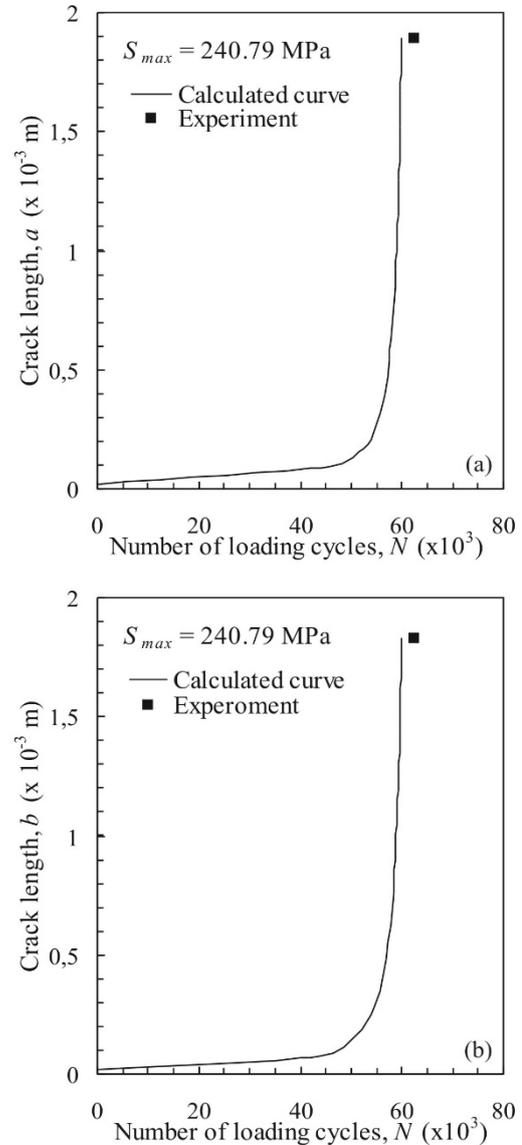


Fig. 3 Fatigue life evaluation for the plate with a quarter-elliptical crack ($S_{max}=240.79$ MPa): (a) $a-N$ and (b) $b-N$ (calculated curves are the present results and experiments are reported in Ref. [12])

Fatigue life and stress intensity factor which are evaluated as a function of crack length in depth and surface direction respectively, are shown in Fig. 4a, 4b and Fig. 5a, 5b for three values of the stress ratio, assuming the same material parameters as those mentioned in Section 4.1. Moreover, the stress raiser analysis examines the fatigue behaviour of three plates ($w=60$ mm, $a_0=b_0=1.85$ mm, $S_{max}=250$ MPa and $R=0.1$), made of 2024 T3 aluminium alloy. Such plates with a quarter-elliptical crack are characterized by the following thicknesses: $t=6$ mm, 7.5 mm and 9 mm, respectively. Relevant fatigue evaluations for the number of loading cycles, as a function of crack length in depth and surface direction, are shown in Fig. 6a and 6b, respectively.

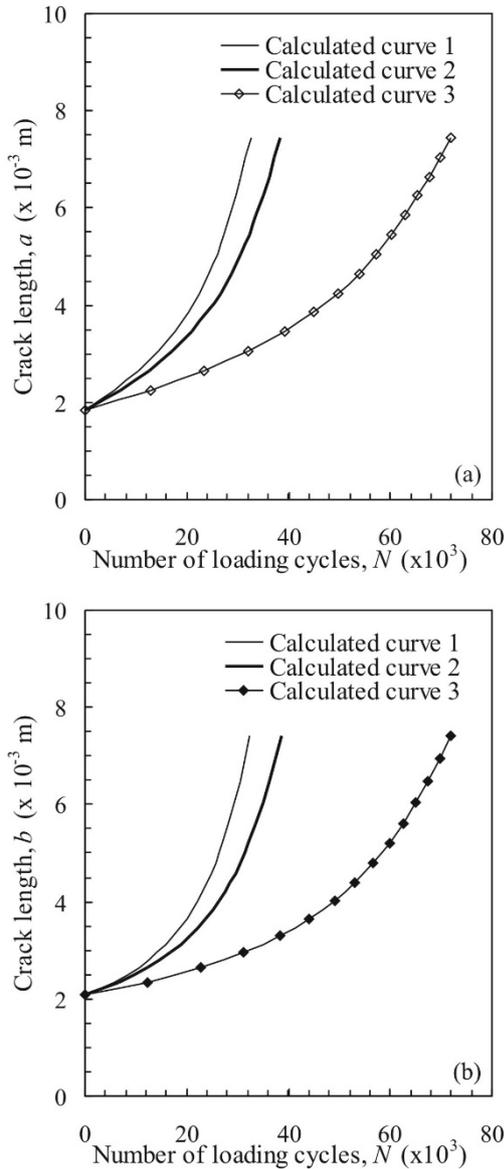


Fig.4 Fatigue life evaluation for the plate with a quarter-elliptical crack ($S_{max}= 300$ MPa): (a) $a - N$ and (b) $b - N$ ($1 - R = 0.1$, $2 - R = 0.25$, $3 - R = 0.625$, calculated curves are the present results)

From theoretical outcomes presented in Fig. 2 to 5, it can be deduced that the effects of applied loads can seriously compromise the structural durability under cyclic loading. Also, relevant comparisons shown in Fig. 6 indicate that the thickness is a factor that can have an important impact on the safe functioning of plate-type systems.

5. CONCLUSIONS

Improving the understanding of surface flaw mechanisms is a relevant task that can prevent the onset of sudden fatigue failure in large moving systems. Therefore, in the present research work a novel analytical model is proposed, in which the Walker's crack growth model is extended to quantify the progression of surface stress-raiser due to cyclic loading.

Furthermore, novel relationships for the life assessment are presented, and the estimated results demonstrate that the developed computational tool can be successfully applied for evaluating the fatigue stability of plate-type structural components with a quarter-elliptical flaw.

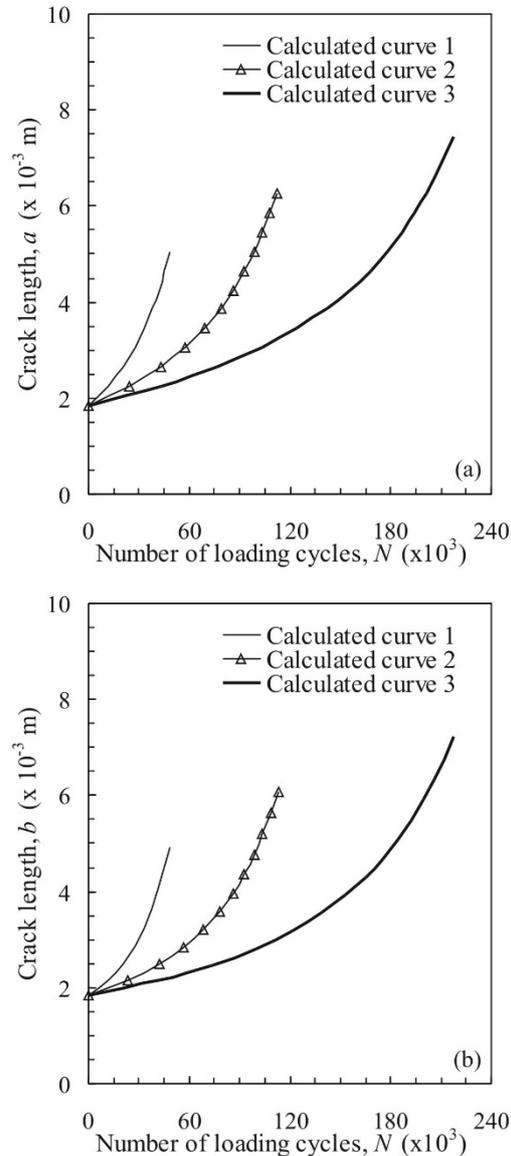


Fig. 6. Fatigue life evaluation for the plate with a quarter-elliptical crack ($S_{max}= 250$ MPa): (a) $a - N$ and (b) $b - N$ ($1 - t = 6$ mm, $2 - t = 7.5$ mm, $3 - t = 9$ mm, calculated curves are the present results)

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NOTE

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